ARithmetische **G**eometrie **O**ber**S**eminar

Overconvergence and classicity

Wintersemester 2011/12

In this Seminar we will study criteria when overconvergent p-adic modular forms are classical, following the papers [PS1] and [PS2] of Pilloni and Stroh. The modular forms appearing in this context are generalizations of elliptic p-adic modular forms and their overconvergent variants, to the case of the Hilbert-Blumenthal moduli problem, resp. to the case of Shimura varieties of type A and C. In the case of elliptic modular forms (i.e. the group GL_2) it is a result of Coleman that overconvergent modular forms of integral weight and small slope are classical:

Theorem [Co]: *An overconvergent elliptic p-adic modular form f of weight $k \in \mathbb{Z}$ and slope* $\mathrm{val}_p(a_p) < k - 1$ *is classical.*

The aim of the first part of the seminar is to prove a similar result for Hilbert modular forms, see [PS1, Theorem 1.1]. In the second part we will develop the theory for modular forms on Shimura vareties of type A and C and give a proof of the classicity criterion for type A.

1) p-adic modular forms
Recall the definition of classical modular forms and show that modular forms are sections of a certain line bundle; then state the q-expansion principle, see [Ka, 1.2-1.7] for example. Give Serre's definition of p-adic modular forms (of level 1) as p-adic limits of Fourier expansions [CL, 1.1] and define the Hecke operators [CL, 1.2]. Now define p-adic modular forms à la Katz for arbitrary level, see [CL, 2.1,2.2] and their weights [CL, 2.4], see also [Ka, 2]. Define the spaces of overconvergent modular forms of integral weight [CL, 5.2] and give their characterization as sections of a line bundle on a rigid analytic space. Finally define the Hecke operators on p-adic modular forms [CL, 2.3] and on overconvergent p-adic modular forms [CL, 5.4] .

2) p-adic Hilbert modular forms
Define the Hilbert-Blumenthal moduli problem [PS1, 3.1] and the spaces of Hilbert modular forms [PS1, 5.1]. Recall the classification of Raynaud-schemes [Ray, 1.4 and Corollary 1.5.2] and describe the partial order on Raynaud-schemes [PS1, 3.2.2], see also [Ray, 2.2]. Then define the degree functions in [PS1, 3.2.4] and the spaces $X_{\geq(v_i)}$ and $X_{\geq v}$. Finally define the Hecke correspondences [PS1, 4.1] and study their action on the strata defined before, see [PS1, 4.2].

3) Hecke operators and the classicity criterion for overconvergent Hibert modular forms
Prove the results on overconvergence of the special subgroups [PS1, 4.3]. Define the Hecke operator [PS1, 5.2.1] and calculate the norm of its special part [PS1, 5.2.2]. Finally prove the main result [PS1, Theorem 6.1].

4) Modular forms on Shimura varieties of type A and C
Define the Shimura varieties of type A and C as in [PS2, 1.1-1.3], see also [Ko]. Then define the classical modular forms [PS2, 1.4] and their overconvergent variants [PS2, 1.5] in analogy with the GL_2-case.

5) Finite flat group schemes and canonical subgroups à la Fargues
Develop the theory of the degree of finite flat group schemes [Fa1, 3] as far as needed to prove [Fa1, Corollary 3]. Then define the canonical subgroup, see [Fa2, Theorem 6] for example, and show that it overconverges [Fa2, Theorem 8 (1)]. Finally discuss the dynamics of the Hecke-operator, see [PS2, 3].

6) Kottwitz-Rapoport strata and the structure of the special fiber
Recall the Kottwitz-Rapoport stratification of local models [PS2, 4.1] and [KR]. Whenever necessary we restrict to type A in this talk. Then define the KR-strata on the stack $\mathrm{BT}_{l,m,\mathrm{Iw}}$, see [PS2, 4.1]. Prove Corollary 4.2.6 and Proposition 4.2.8 of *loc. cit.* Finally define the subschemes W^0 and W^1, see [PS2, 4.3.1].

7) The decomposition of the Hecke operator
First prove [PS2, Prop. 4.3.2.6 and Lemma 5.1.2] and the decomposition of the Hecke correspondences [PS2, 5.3]. Then discuss the resulting decomposition of the Hecke operator [PS2, 5.4].

8) The classicity criterion for type A
Prove the estimates on so-called Kassei-series [PS2, 5.5]. Then prove the main result [PS2, Corollary 5.6.2] on the extension of overconvergent modular forms in case A. Then sketch the material presented in [PS2, 6], as far as needed to deduce the classicity criterion [PS2, Theorem 6.3.1] for Shimura varieties of type A.

References

[CL] A. Chambert-Loir, *Formes modulaires p-adiques*, Modular forms and Galois representations, Luminy 1997. Available at http://arxiv.org/abs/math/9712297.

[Co] R. F. Coleman, *Classical and overconvergent modular forms*, Inv. Math. **127** no.3 (1997).

[Fa1] L. Fargues, *La filtration de Harder-Narasimhan des schémas en groups finis et plats*, J. Reine Angew. Math. **645** (2010).

[Fa2] L. Fargues, *La filtration canonique des points de torsion des groupes p-divisibles*, avec la collaboration de Yichao Tian, to appear in Ann. Sci. École Norm. Sup. Available at http://www.math.u-psud.fr/ fargues/Prepublications.html.

[PS1] V. Pilloni, B. Stroh, *Surconvergence et classicite: Le cas Hilbert*, preprint 2010. Available at http://www.math.univ-paris13.fr/~stroh.

[PS2] V. Pilloni, B. Stroh, *Surconvergence et classicite: Le cas deploye*, preprint 2010. Available at http://www.math.univ-paris13.fr/~stroh.

[Ka] N. Katz, *p-adic properties of modular schemes and modular forms*, in Modular functions of one variable III, LNM 350, p. 69-190, Springer-Verlag.

[Ko] R. Kottwitz, *Points on Shimura varieties over finite fields*, J. Amer. Math. Soc. **5** (1992).

[KR] R. Kottwitz, M. Rapoport, *Miniscule alcoves for* GL_n *and* GSp_{2n}, Manuscripta Math. **102** (2000).

[Ray] M. Raynaud, *Schémas en groupes de type* $(p, p, \ldots, p)$, Bull. Soc. Math. de France **102** (1974).